Next-to-leading prediction of ϵ'/ϵ: an upgraded analysis

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Abstract

We present an updated theoretical prediction of ϵ'/ϵ, using the next-to-leading $\Delta S=1$ effective hamiltonian and lattice QCD matrix elements. The CP violating phase is costrained by using both the experimental values of ϵ and x_d, assuming the theoretical determination of f_B. Predictions of $\cos\delta$ and $\sin 2\beta$ are also obtained in this way. For ϵ'/ϵ, our estimate is $\epsilon'/\epsilon=(2.8\pm 2.4)\times 10^{-4}$.

We repeat the combined analysis of the CP violation parameter ϵ and the B-mixing parameter x_d in order to estimate ϵ'/ϵ, along the lines followed in refs. [1, 2]. The main steps of this analysis are the following:

1 The CP violating phase δ of the CKM matrix is constrained by comparing the theoretical prediction for ϵ with its experimental value. To this purpose, the relevant formula is

$$\begin{aligned} |\epsilon|_{\xi=0} &= C_\epsilon B_K A^2\lambda^6\sigma\sin\delta\,\{F(x_c,x_t)+ \\ &\quad F(x_t)[A^2\lambda^4(1-\sigma\cos\delta)]-F(x_c)\}\,, \end{aligned} \tag{1}$$

where $x_q=m_q^2/M_W^2$ and the functions $F(x_i)$ and $F(x_i,x_j)$ are the so-called *Inami-Lim* functions [3], obtained from the calculation of the basic box-diagram and including QCD corrections. $F(x_t)$ is known at the next-to-leading order, which has been included in our calculation [4]. In eq. (1),

$$C_\epsilon=\frac{G_F^2 f_K^2 m_K M_W^2}{6\sqrt{2}\pi^2\Delta M_K}, \tag{2}$$

where ΔM_K is the mass difference between the two neutral kaon mass eigenstates. Moreover, $\rho=\sigma\cos\delta$ and $\eta=\sigma\sin\delta$, where λ, A, ρ and η are the parameters of the CKM matrix in the Wolfenstein parametrization [5]. Finally, B_K is the renormalization group invariant B-factor [6].

2 The theoretical estimate of the B-meson coupling constant f_B is used to further constrain δ, by comparing the theoretical prediction of the B mixing parameter x_d with its experimental value. From the $\Delta B=2$ effective hamiltonian, one can derive

$$\begin{aligned} x_d &= \frac{\Delta M}{\Gamma}=C_B\frac{\tau_B f_B^2}{M_B}B_B A^2\lambda^6\Big(1+ \\ &\qquad \sigma^2-2\sigma cos\delta\Big)F(x_t), \\ C_B &= \frac{G_F^2 M_W^2 M_B^2}{6\pi^2}, \end{aligned} \tag{3}$$

where B_B is the B-parameter relevant for $B-\bar{B}$ matrix element

$$\langle\bar{B}_d|(\bar{d}\gamma_L^\mu b)^2|B_d\rangle=8/3 f_B^2 M_B^2 B_B. \tag{4}$$

Notice that $f_B B_B^{1/2}$ must be known, for the experimental value of x_d to give a constraint on δ.

3 From the $\Delta S=1$ effective hamiltonian, one can calculate the expression of ϵ' in terms of CKM matrix elements, Wilson coefficients and local operator matrix elements. One has

$$\epsilon'=\frac{e^{i\pi/4}}{\sqrt{2}}\frac{\omega}{\mathrm{Re}A_0}\left[\omega^{-1}(\mathrm{Im}A_2)'-(1-\Omega_{IB})\,\mathrm{Im}A_0\right], \tag{5}$$

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where $(\mathrm{Im}A_2)'$ and $\mathrm{Im}A_0$ are given by

$$\begin{aligned}
\mathrm{Im}A_0 &= -G_F Im\left(V_{ts}^* V_{td}\right)\{-(C_6B_6+ \\
&\quad \frac{1}{3}C_5B_5\Big) Z + \left(C_4B_4 + \frac{1}{3}C_3B_3\right) X + \\
&\quad C_7B_7^{1/2}\left(\frac{2Y}{3} + \frac{Z}{6} + \frac{X}{2}\right) + \\
&\quad C_8B_8^{1/2}\left(2Y + \frac{Z}{2} + \frac{X}{6}\right) - \\
&\quad C_9B_9^{1/2}\frac{X}{3} + \left(\frac{C_1B_1^c}{3} + C_2B_2^c\right) X\Big\},
\end{aligned} \tag{6}$$

and

$$\begin{aligned}
(\mathrm{Im}A_2)' &= -G_F Im\left(V_{ts}^* V_{td}\right)\Big\{C_7B_7^{3/2}\Big(\frac{Y}{3} - \\
&\quad \frac{X}{2}\Big) + C_8B_8^{3/2}\left(Y - \frac{X}{6}\right) + \\
&\quad C_9B_9^{3/2}\frac{2X}{3}\Big\}.
\end{aligned} \tag{7}$$

Here $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0$ and we have introduced $(\mathrm{Im}A_2)'$ defined as

$$\mathrm{Im}A_2 = (\mathrm{Im}A_2)' + \Omega_{IB}(\omega \mathrm{Im}A_0). \tag{8}$$

Ω_{IB} accounts for the isospin breaking contribution, see for example ref. [7]. The Wilson coefficients C_i have been evaluated at the next-to-leading order for $\mu = 2$ GeV, using the anomalous dimension matrices given in refs. [8, 9] and the initial conditions computed in refs. [10, 11] (given for HV in ref. [12]). Concerning the local operator matrix elements, their values are given by a set $\{B_i\}$ of B-parameters multiplied by the vacuum insertion approximation results. In turn, these can be written in terms of the three quantities (see eq. (6) and eq. (7))

$$\begin{aligned}
X &= f_\pi\left(M_K^2 - M_\pi^2\right), \\
Y &= f_\pi\left(\frac{M_K^2}{m_s(\mu) + m_d(\mu)}\right)^2 \\
&\sim 12\,X\left(\frac{0.15\,\mathrm{GeV}}{m_s(\mu)}\right)^2, \\
Z &= 4\left(\frac{f_K}{f_\pi} - 1\right) Y.
\end{aligned} \tag{9}$$

The numerical values of the B-parameters have been taken from lattice calculations [15] For those B-factors which have not yet been computed on the lattice, we have used educated guesses, see ref. [13].

More details on this analysis can be found in ref. [13, 14]. Compared to our previous work [2], the main improvements are the following:

Parameters	
$(f_B B_B^{1/2})_{th} = (200 \pm 40)$ MeV	$V_{cb} = A\lambda^2 = 0.040 \pm 0.006$
$m_s(2\mathrm{GeV}) = (128 \pm 18)$ MeV	$x_d = 0.685 \pm 0.076$
$\Lambda_{QCD}^{n_f=4} = (330 \pm 100)$ MeV	$\Omega_{IB} = 0.25 \pm 0.10$
$\tau_B = (1.49 \pm 0.12) \times 10^{-12}$ s	$m_t = (174 \pm 17)$ GeV
$\lvert V_{ub}/V_{cb}\rvert = \lambda\sigma = 0.080 \pm 0.015$	

Table 1. Values of the fluctuating parameters used in the numerical analysis.

1 The constraint on δ coming from x_d is used in the analysis, taking f_B from the theory. Since there is increasing theoretical evidence that the value of f_B is large ($\sim$ 200 MeV) and that the relevant B-parameter B_B is close to one, this constraint is quite effective.

2 Updated values of the experimental parameters entering in the phenomenological analysis, such as the B meson lifetime τ_B, the B_d^0–$\bar{B}_d^0$ mixing parameter x_d, the CKM matrix elements, ($|V_{cb}|$, $|V_{ub}|/|V_{cb}|$), etc., have been used.

3 The value of the strange quark mass m_s has been taken from lattice calculations [16], thus making a more consistent use of lattice results for the B-parameters of the relevant penguin operators.

4 All the results are presented with an estimate of the corresponding errors. These errors come from the limited precision of measured quantities, e.g. τ_B, and from theoretical uncertainties, e.g. the values of hadronic matrix elements.

The results of our analysis have been obtained by varying the experimental quantities, e.g. the value of the top mass m_t, τ_B, etc. and the theoretical parameters, e.g. the B-parameters, the strange quark mass $m_s(\mu)$, etc., according to their errors. Values and errors of the input quantities used in the following are reported in tables 1–3. We assume a gaussian distribution for the experimental quantities and a flat distribution (with a width of 2σ) for the theoretical ones. The only exception is $m_s(\mu)$, taken from quenched lattice QCD calculations, for which we have assumed a gaussian distribution, according to the results of ref. [16].

The theoretical predictions ($\cos\delta$, $\sin 2\beta$, ϵ'/ϵ, etc.) depend on several fluctuating parameters. We have obtained numerically their distributions, from which we have calculated the central values and the errors reported below.

Using the values given in the tables and the formulae given previously, we have obtained the following results:

1 The distribution for $\cos\delta$, obtained by comparing the experimental value of ϵ to its theoretical

Figure 1. Distributions of values for $\cos\delta$, $\sin 2\beta$ and ϵ'/ϵ, using the values of the parameters given in the tables. The solid istograms are obtained without using the x_d constraint. The dashed ones use this constraint, assuming that $f_B B_B^{1/2} = 200 \pm 40$ MeV. The contour-plot of the event distribution in the $\rho - \eta$ plane is also shown.

Constants	
$G_F = 1.16634 \times 10^{-5}\mathrm{GeV}^{-2}$	$f_\pi = 132$ MeV
$m_c = 1.5$ GeV	$f_K = 160$ MeV
$m_b = 4.5$ GeV	$\lambda = \sin\theta_c = 0.221$
$M_W = 80.6$ GeV	$\epsilon_{exp} = 2.268 \times 10^{-3}$
$M_\pi = 140$ MeV	$\mathrm{Re}A_0 = 2.7 \times 10^{-7}$ GeV
$M_K = 490$ MeV	$\omega = 0.045$
$M_B = 5.278$ GeV	$\mu = 2$ GeV
$\Delta M_K = 3.521 \times 10^{-12}$ MeV	

Table 2. Values of the constants used in the numerical analysis.

prediction, is given in figure 1. As already noticed in refs. [1, 2] and [17, 18], large values of f_B and m_t favour $\cos\delta > 0$, given the current measurement of x_d. When the condition 160 MeV $\leq f_B B_B^{1/2} \leq 240$ MeV is imposed (f_B-cut), most of the negative solutions disappear, giving the dashed istogram of figure 1, from which we estimate

$$\cos\delta = 0.47 \pm 0.32 \ . \tag{10}$$

2 The value of $\sin 2\beta$ depends on $\cos\delta$. The distribution of $\sin 2\beta$ is shown in figure 1, without (solid) and with (dashed) the f_B-cut. When the f_B-cut is imposed, one gets larger values of $\sin 2\beta$ [1]. From the dashed distribution, we obtain

$$\sin 2\beta = 0.65 \pm 0.12 \ . \tag{11}$$

Figure 1 also contains the contour-plot of the event distribution in the $\rho - \eta$ plane, showing the effect of the ϵ and x_d constraints, when the f_B-cut is imposed.

3 In figure 2, several informations on ϵ'/ϵ are provided. Contour-plots of the distribution of the generated events in the ϵ'/ϵ–$\cos\delta$ plane are shown, without and with the f_B-cut. One notices a very mild dependence of ϵ'/ϵ on $\cos\delta$. As a consequence one obtains approximatively the same prediction in the two cases (see also figure 1)

$$\epsilon'/\epsilon = (2.3 \pm 2.1) \times 10^{-4} \ \ \mathrm{no-cut}, \tag{12}$$

and

$$\epsilon'/\epsilon = (2.8 \pm 2.4) \times 10^{-4} \ \ f_B - \mathrm{cut}. \tag{13}$$

In figure 2, we also give ϵ'/ϵ as a function of m_t. The band corresponds to the 2σ prediction.

In spite of several differences, the bulk of our results overlap with those of ref. [19]. It is reassuring that theoretical predictions, obtained by using different approaches to evaluate the operator matrix elements, are in good agreement.

Figure 2. Above, contour-plots of the event distributions in the plane ϵ'/ϵ–$\cos\delta$ without and with the f_B-cut. Below, ϵ'/ϵ as a function of m_t.

B-parameters	
$B_K = 0.75 \pm 0.15$	$B_9^{(3/2)} = 0.62 \pm 0.10$
$B^c_{1-2} = 0 - 0.15^{(*)}$	$B_{3,4} = 1 - 6^{(*)}$
$B_{5,6} = B_{7-8}^{(3/2)} = 1.0 \pm 0.2$	$B_{7-8-9}^{(1/2)} = 1^{(*)}$

Table 3. Values of the B-parameters, for operators renormalized at the scale $\mu = 2$ GeV. The only exception is B_K, which is the RG invariant B-parameter. $B_9^{3/2}$ has been taken equal to B_K, at any scale. The value reported in the table is $B_9^{3/2}(\mu = 2\text{GeV})$. Entries with a $^{(*)}$ are educated guesses, the others are taken from lattice QCD calculations.

On the basis of the latest analyses, it seems very difficult for ϵ'/ϵ to be larger than 10×10^{-4}. This may happen by taking the matrix elements of the dominant operators, Q_6 and Q_8, much different than usually assumed. One possibility, discussed in ref. [19], is to take $B_6 \sim 2$ and $B_8 \sim 1$, instead of the usual values $B_6 \sim B_8 \sim 1$. To our knowledge, no coherent theoretical approach can accomodate so large value of B_6.

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